Systematical measurement errors

Along the lines of the rule formulated by Schrödinger that a system can influenced even by observing, an EUT can be influenced by a normal measurements. If the measurement problem is not elaborated carefully, avoidable measurement errors may occure causing an incorrect result. Connecting the power analyzer to the EUT causes an alternation of the complete circuit. These changes implicate so-called “systematic measurement errors”. Determining the error source can avoid or correct incorrect measurements in an easy way.

Crucial for the usability of the measurement is whether the reliability of the results is within the given tolerances. The following examples will show you why systematic errors occur, which magnitude they have and how they can be avoided. All measurements within this application note were performed with a precision power analyzer LMG450 engineered and manufactured by ZES ZIMMER Electronic Systems GmbH, Germany. All spectra and screenshots are taken from this device.

Test circuits

Representation of the voltages of a bridge rectifier

As an introduction of the topic, a very simple bridge rectifier circuit with one smoothing capacitor and one load resistor shall be contemplated. Input voltage, is a pure sinusoidal signal with a frequency of 50Hz and a root-mean-square value of 230V. The source is connected to earth potential on one side, just like in the normal low voltage network. For these basic considerations, the output voltage (see Figure 2) at the load resistor (see Figure 1) was simulated by a PC and can be verified by a simple measurement setup.

![Figure 1: Rectifier circuit](image-url)
The result shown in the figure above appears as expected. There is a DC voltage of 324V with a ripple voltage which can be traced back to the discharge of the capacitor ($v(plus) - v(minus)$).

Now we take a look at the voltages $v(plus)$ and $v(minus)$ referred to earth potential. As visible, two DC voltages of $\pm 160V$ with a superposed 50Hz AC voltage with a RMS value of $\approx 114Vrms$ can be observed.

Please note that the signals $v(plus)$ and $v(minus)$ are no pure sinusoidal voltages. The disturbance of the sinusoidal waveform is caused by the threshold voltage of the diodes and the noise (load of the resistor). If there are additional disturbances at the line voltage (e.g. harmonics) the sinusoidal waveform will suffer a lot more interference (high frequency harmonics).

**Contemplation of the voltages at a bridge rectifier circuit without smoothing**

In the next step we analyze the same circuit without the smoothing capacitor. The input voltage shall be the same as in the previous example (waveform refer Figure 4).

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**Figure 2: The simulated voltages with a smoothing capacitor**

**Figure 3: Rectifier circuit without smoothing**
The output voltage is nearly the absolute value of the 50Hz input voltage (just disturbed by the threshold voltage and the noise) because there is no energy storing element in the circuit that could buffer the output voltage.

Again, we take a closer look at $v(\text{plus})$ and $v(\text{minus})$ referred earth. In both cases, the voltage is a one way rectified waveform with a frequency of 50Hz. Because of the y-axis scaling, the threshold voltage of the diode is not recognisable.

Conclusion: In each circuit, we were able to obtain an AC voltage against earth. The fundamental frequency of this voltage is 50 Hz, but the harmonic content depends on the load and the distortion of the input voltage.
Three phase contemplation: Three phase rectifier

The contemplations of the previous examples should now be analysed in a three phase system. The input voltages with a phase-shift of 120° are pure sinusoidal.

Figure 5: Three phase rectifier circuit

First we recognize that the output voltage between v(plusdreh)-v(minusdreh) is a DC voltage of approximately 550 V with a superposed AC voltage of about 10 Vpk and a frequency of about 300 Hz.

If you take a look at the output voltages v(plusdreh) and v(minusdreh) separately and referred to earth, you can perceive a DC voltage of 275 V with -contrariwise polarity to each other- which is superposed by a 150Hz oscillation of about 35Vpk.

Figure 6: The simulated voltages with smoothing capacitor
In a three phase system, the voltage referred to earth has the threefold frequency with a shorter amplitude.

Figure 7: Three phase rectifier without smoothing
As in the single-phase example above, we shall have a closer look at the circuit without the smoothing capacitor. The voltage-waveforms will look like this:

Figure 8: Voltage curves without smoothing capacitor in a three phase system

Again, the output voltage \(v(\text{plusdreh})-v(\text{minusdreh})\) consists of a ca. 550V DC voltage superposed by a 300Hz oscillation with ca. 40Vpk.

The separate output-voltages \(v(\text{plusdreh})\) and \(v(\text{minusdreh})\) are also built by a 275V DC voltage (with reversed signs) which is superposed by an AC voltage with a frequency of 150Hz and an amplitude of about 100V.

Connection of a voltage measurement channel

The voltage channel of a measurement device can be described by the following equivalent circuits:

Figure 9: Voltage measurement channel
Typical values for the components in this:

- \( R_{\text{mess}} = 1 \text{ M}\Omega \ldots 10 \text{ M}\Omega \)
- \( C_i = 12 \text{ pF} \ldots 23 \text{ pF} \ldots 50 \text{ pF} \)
- \( C_{\text{earth}} = 50 \text{ pF} \ldots 300 \text{ pF} \).

The input capacitor \( C_i \) can be neglected. Because of the following consideration it will take the impedance-value of:

\[
X_{ci} = \frac{1}{2\pi \cdot 50Hz \cdot 10 \cdot 10^{-12} F} = 318.30 \text{ M}\Omega
\]

In the parallel connection this resistor has hardly no influence. Usually, the measurement connection looks like the circuit in figure 10:

![Figure 10: "Normal" connection of a measuring voltage channel](image)

In this wiring, the systematic measurement error is reduced to an insignificant minimum, because the measured voltage drop only appears across \( R_{\text{mess}} \). The current \( I_{\text{earth}} \) against earth, caused by the superposed sinusoidal input components at the + pole, will take the way through \( C_i \) and \( C_{\text{earth}} \), because the capacitor has an impedance of ca. 318\( \Omega \) at 50Hz. The sinusoidal component at the "-" pole will take the way directly through \( C_{\text{earth}} \) against earth and the voltage at \( C_{\text{earth}} \) does not influence the result.

Now the measurement channel is connected in the following way:
Hence, the AC-current, driven by the output voltage through $C_{\text{earth}}$, generates an additional voltage drop referred to earth at $R_{\text{mess}}$. In turn, this earth current causes the meter to show a signal with a fundamental frequency of 50 Hz. This wiring seems to be illogical at the first glance, but it makes sense in the following background: The next example will show that if a measuring system has more than two output connectors, the measurement according to figure 10 will be rarely applicable.

**Measurement at a frequency converter**

A frequency converter usually consists of a rectifier stage (one or three phase) followed by an inverter module (in the example pulse width modulation). The following equivalent circuit represents the frequency converter:

![Equivalent circuit of a frequency converter](image)

The rectified voltage (the effects are similar for one and three phase operation) will be switched to the motor winding with a defined cycle ratio. The sequence of this pulses results in the RMS value of the AC voltage with the desired fundamental frequency.
In this application, not only the fundamental frequency, but also the superposed voltage components mentioned in the beginning are connected to the motor windings. Anyhow, that does not affect the motor because the influence is equal at all three phases. Any measurement at the output of this system will cause an earth current due to the earth capacitor of the measurement device with a systematic measurement error as inevitable consequence.

In this example, a single phase rectifier is applied so we expect a 50Hz component. There is more than one possibility to connect the measuring channels to the circuit and each one has its advantages and disadvantages as described in the following section. Depending on the measurement problem, you can minimize the systematic measurement errors by choosing the right wiring for your application. The selection of the wiring is very important.

**Remark:** In all following examples, the motor is driven with a frequency of 10Hz. For a better readability of the spectra, three inter-harmonics were measured additionally (except at delta- and aron-wiring). Thus, the fundamental of the motor is located at the $4^{th}$ harmonic which is marked by the cursor. The root-mean-square value is indicated on the left side of the screen-shot. In the reckoning, we only calculate with the value of the first phase, because the spectra are equal and therefore the effects are equal at all three phases. The superposed 50Hz line-noise is the $20^{th}$ harmonic and the $3^{rd}$ harmonic of the line voltage the $60^{th}$ (150Hz). These harmonics are also marked in the second screen-shot and the amplitude values can be read on the left and the right of the belonging figure.

**Direct wiring without artificial midpoint**

The motor was operated in the following wiring:

![Diagram](image)

Figure 13: Direct connection of the measuring channels to the motor
Evaluation of the spectra

As you can see very clearly, the 20\textsuperscript{th} harmonic (50 Hz oscillation) contributes a lot towards the RMS value in comparison to the motors fundamental (10Hz). This causes a grave systematic measurement error which can be calculated in the following way:

\[ \sqrt{U_{\text{motor}}^2 + U_{50Hz}^2} = 26.3V + 11.08V = 28.54V \]

Total RMS of the 10Hz signal and the 50Hz component

\[ 28.54V - 26.3V = 2.24V \]

Absolute error

\[ \frac{2.24V}{26.3V} = 0.085 \]

Relative error

which equates to 8.5 \% error at the RMS value.

Additionally, the error caused by the 3\textsuperscript{rd} harmonic will also be calculated with the square value. The impact of this wiring: The frequency converter is only little loaded resistively or capacitively, related to the motor values. The merrier the difference between the characteristics of the measurement channel and the motor is, the more this effect will preponderate.
**Connection via an artificial midpoint**

As a common method, the artificial midpoint has approved its worth for the measurement of frequency converters, especially if a meter without star/delta conversion is applied. The wiring with artificial midpoint means that a 68 kΩ resistor is connected in parallel to the voltage measurement channel (input impedance 1 MΩ). In this case, 90% of the superposed AC current flows through this resistor against earth and is not measured in the meter channel. Thus, the error is reduced by the same factor.

![Figure 16: Connection of an artificial midpoint](image1)

![Figure 17: Spectrum of the three motor phases wired with an artificial midpoint](image2)
Evaluation of the spectra

By connecting the artificial midpoint, the value of the 20th harmonic (50Hz oscillation) is divided by the factor of about 15. For the calculation of the measurement error the result is:

\[
\sqrt{26.2^2 + 0.7379^2} = 26.21V \\
26.21V - 26.2V = 0.01V \\
\frac{0.01V}{26.2V} = 0.00038
\]

which gives an error of 0.038 %.

As you can see, the error caused by the superposed line voltage is decreased by a factor of about 220. According to the expected result of $15^2 = 225$ (because the error is calculated with the squared values), this value is plausible.

Delta wiring

Now, the voltage measurement channels are connected in delta wiring and the current measurement channels in star wiring. For this method, you require a modern power analyzer like the LMG450 or LMG500, which comes with the star/delta conversion as an option. To measure in this mode you must switch the wiring to $U\Delta I^* \rightarrow U^* I^*$. The advantage of this connection lies in the fact that at each phase an earth capacity is connected and the disturbances will take their way through this capacitor and not via the measurement resistor. By the built-in star/delta conversion all values are available for displaying.

In the circuit diagram below you can see the optimal wiring for the voltage channels. If you do not care about the shown connection, the indexes of the separate calculated values will not correspond.
Figure 19: Delta wiring

Figure 20: Spectra of the motor phases in delta wiring

Figure 21: Spectrum of the first motor phase in delta wiring
**Evaluation of the spectra**

For the delta connection, you can estimate the following measurement error:

\[
\sqrt{26.42^2 + 0.1139^2} = 26.420246\ V
\]

\[
26.420246\ V - 26.420246\ V = 0.000246\ V
\]

\[
\frac{0.000246\ V}{26.42\ V} = 0.0000929
\]

what equals an error of 0.000929%.

By using this very simple method, the systematic error can be reduced extremely. Impact of this wiring in a nutshell: small ohmic load, high capacitive load.

**Aron wiring**

The wiring shown below is called “aron wiring” and has the advantage that you only need two measurement channels to determine the active power.

With a three phase meter, you can additionally measure the input active power with the third channel (e.g. one phase frequency converter). With a four channel power analyzer like the LMG450 you can even measure two three phase systems at the same time. A very important advantage of this power analyzer with integrated star/delta conversion: it calculates all current, voltage and power allocations of the six phases. You can evaluate **ALL** values of both systems even though the aron wiring is meant for evaluating only the active power. (Wiring: 2+2 UΔI*→U*I*). With the aron wiring, you measure the voltages U₁₃ and U₂₃ and the currents I₁ and I₂. In the example the wiring is in load side voltage measurement mode, because our evaluations are made with the voltages close to the motor. As you can see in “Figure 22: Aron wiring“ two capacitors are connected to the third phase. The very good result of the measurement is made, because the superposed 50Hz component flows on the one hand through the motor windings and on the other hand through the switches of the frequency converter over the third phase against earth. The both impedances are much lower than the 1MΩ channel resistor. Disadvantageous about the aron wiring is that even at small loads the circuit can be loaded unsymmetrically.
Figure 22: Aron wiring

Figure 23: Spectra of the motor phases in aron wiring

Figure 24: Spectrum of the first motor phase in aron wiring
**Evaluation of the spectra**

For the aron wiring the error is calculated:

\[
\sqrt{26.61^2 + 0.1116^2} = 26.61023402V
\]

\[
26.61023402V - 26.61V = 0.00023402V
\]

\[
\frac{0.00023402V}{26.61V} = 0.0000879
\]

that equals an error of 0.000879 %

You see in aron wiring the measurement error is very small, too.

Impact of this wiring: small ohmic load, medium capacitive load

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**Isolating transformer**

Due to the galvanic isolation the current flow against earth can be suppressed almost completely, because almost no current can flow though the capacitive coupling of the isolating transformer.

**Remark:** Refer to the safety instructions weather the frequency converter is permitted to be used without a protective conductor!

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*Figure 25: Wiring with an isolating transformer*
Figure 26: Spectra of the motor phases wired via an isolating transformer

Figure 27: Spectrum of the first motor phase

**Evaluation of the spectra**

For the wiring via an isolated transformer the error is calculated:

$$\sqrt{26.51^2 + 1.367^2} = 26.545V$$

$$26.545V - 26.51V = 0.03522V$$

$$\frac{0.03522V}{26.51V} = 0.00133$$

gives 0.133% error.

Inserting an isolated transformer can reduce the systematic errors, too. Furthermore the converter is only loaded with the motor and no longer with the earth capacitors of the meter.

**Hand held meters (battery operation)**

Showing the effects of the earth capacitors in a very impressive way is to measure with hand held meters in the above shown connection (Figure 11). Depending on the actual position of the meter the measurement results vary. The following table shows the RMS measurement results of a rectifier circuit with different positions of the meter (load: 1MΩ input voltage 230V):
Position of the meter | $v_{\text{plus}}$, $v_{\text{minus}}$ open | $v_{\text{minus}}$, $v_{\text{plus}}$ open
--- | --- | ---
10cm high on a carton | 2.75V | 2.95V
directly on an earthed steel plate | 6.01V | 6.24V

These large variations can be explained with the „undefined“ way to the earth, only. If the meter is placed on an earthed steel plate the AC component influences the measurement resistor more than if it is hand-held.

Conclusion: Power analyzers with an undefined grounding bear the risk of influencing the measurement results. In the example a Fluke73III was used.

Connecting a current meter
In the examples above we saw, that selecting the correct connection of the meter to the circuit can greatly increase the accuracy. All considerations made for the voltage channels should be analysed for the connection of the current channels, because there are similar effects.

The current channel of a meter is represented by the following equivalent circuit:

![Figure 28: Equivalent circuit of a current channel](image)

By connecting the frequency converter to the I* socket (motor to I) an additional current will flow through the measurement shunt and $C_{\text{earth}}$ against earth and will influence the measurement.
The difference between measuring voltage or current in the correct way
As you could see in the examples above the way of connecting the meter to the circuit may cause large differences in the measurement results. Measuring power an additional factor must be taken into consideration, the influence of the measurement channels among each other.
The different ways of wiring are shown in the figures below.

![Diagram of Measuring voltage in the correct way](image)

**Figure 29: Measuring voltage in the correct way**

In this wiring the voltage channel is not influenced by the impedance of the current channel. On the other hand the current channel additionally measures the current which flows through the voltage channel. That means there is an error in the measurement result of the current caused by the voltage channel. Beside this there is a current flowing through $C_{earth}$ against earth, too.
Figure 30: Measuring current in the correct way

In this wiring the current is measured in the correct way, because the current which flows through the load flows also via the current channel shunt. Just the current which flows through $C_{\text{earth}}$ to earth causes an additional error. On the other hand the voltage channel measures the voltage which drops at the $R_{\text{shunt}}$ of the current channel additional to the voltage at the load.

Consider that all inspections are made from the load side. If you want to measure the output power of the generator all considerations have to be made in the reverse way.

All systematic measurement errors can be terminated by error calculations. Measuring high currents the best way is to measure the voltage correct way, because the high voltage drop of the current channel is not measured additionally.

For high voltages the best way is the current correct way, because the current flow caused by the high voltages in the voltage channel is not measured.

**Conclusion**

The considerations made in this article show that you can influence the accuracy of a measurement result in a high degree. You should use all possibilities of a modern power analyzer like the LMG50 provides to minimize the measurement errors. There is no general answer about the „right“ way to measure. All circumstances and the purpose of the measurement should be taken into consideration. ZES ZIMMER will keep improving power analyzers to get closer to the truth.

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